1 Basic Newtonian Physics

1.1 Position and its derivatives

1.1.1 Position as function of time

$$x(t) \qquad or \qquad < x(t), y(t), z(t) >$$

1.1.2 Velocity as function of time

is rate of change of Position with respect to time.

$$v(t) = dx(t)/dt$$

$$or < v_x(t), v_y(t), v_z(t) >$$

$$= d < x(t), y(t), z(t) > /dt$$

$$=< \frac{dx(t)}{dt}, \frac{dy(t)}{dt}, \frac{dz(t)}{dt} >$$

1.1.3 Acceleration as function of time

is rate of change of Velocity with respect to time.

$$a(t) = dv(t)/dt = d^2x(t)/dt^2$$

$$or < a_x(t), a_y(t), a_z(t) >$$

$$=< \frac{d^2x(t)}{dt^2}, \frac{d^2y(t)}{dt^2}, \frac{d^2z(t)}{dt^2} >$$

1.1.4 Units

m (meters) and s (seconds)

Position: m
Velocity: m/s
Acceleration: $m/s/s = \frac{m}{s^2}$

1.2 Force

Force: F = ma (Newton's 2nd Law (with m = constant))
or $\sum F_x = ma_x$

$\sum F_y = ma_y$

$\sum F_z = ma_z$

Units: Newtons (N) = $\frac{kgm}{s^2}$

Acceleration due to gravity, at sea level, on earth, is:

$9.8m/s^2 = g$

Force due to gravity (sea level, earth) is:

$m \cdot g$ Newtons

Frictional Forces:

When stationary, the coefficient of static friction is active:

μ_s

When moving, the coefficient of kinetic (sliding) friction is active:

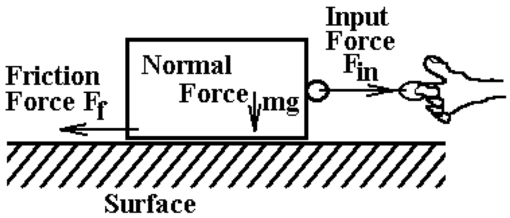
μ_k

Force due to friction acts opposite other forces, and is equal to

$F_N\mu_{(s \quad or \quad k)} = NormalForce \cdot FrictionCoeff$

Example:

$$\sum F = ma = \begin{cases} F_{in} - m_g \cdot \mu_s, & \text{if at rest and} |F_in| < m_g\mu_s \\ F_{in} - m_g \cdot \mu_k, & if F_in > m_g\mu_k \end{cases}$$



1.3 Energy (work) and Energy Flow

Energy: W = F x (force through a distance, if F is constant)

Units: Joules = $Nm = kgm^2/s^2$

Power: = Energy flow, or the time rate of work = $dW(t)/dt$

Units: Watts = Joules / s = kgm^2/s^3

Potential Energy: Potential to do work because of position in a field

In a conservative field, potential (and work) only depends on initial and final positions, not on path or time taken to traverse it.

Example: Gravitational Potential Energy

$= W_g = m \cdot g \cdot h$(h is height)

Kinetic Energy: Potential to do work because of mass in motion

$W_v = 1/2mv^2$

Energy is Conserved, A Very Important Physical Concept!!!

Example: Potential and Kinetic energy, dropping a ball

$$InitialPotential + InitialKinetic$$
$$= FinalPotential + FinalKinetic$$

$m \cdot g \cdot h_i + 0 = 0 + 1/2mv_f^2$

Mass cancels!! (It took a long time to prove this)

Mass is Conserved, Another Very Important Physical Concept!!!

Example: If I pound water into a hose, it either comes out the other end or the hose eventually blows up. Mass is conserved.

1.4 Rotational Frames of Reference

All of the above still works, just adjusted slightly

Force $\rightarrow$ Torque $\tau = F_t r$ (tangential force times radius)

Position $\rightarrow$ Angle θ

Velocity $\rightarrow$ Angular velocity ω

Mass $\rightarrow$ Moment of Inertia I =

$$\begin{cases} mr^2 & \text{for a point mass at distance r} \\ 1/2mr^2 & \text{for a disk about its center} \\ 2/5mr^2 & \text{for a sphere about its central axis} \end{cases}$$

Example: A fulcrum with a pivot at 1/3 its length will balance with twice as much mass on the short end as on the long end.

Assume it's balanced and at rest (acceleration = 0)

$\sum \tau = I0 = 0$ $\qquad\qquad$ $(\sum F = ma = 0)$

so $m_{longend} \cdot g \cdot 2/3 = m_{shortend} \cdot g \cdot 1/3$

thus $2m_{longend} = m_{shortend}$

Rotational Energy:

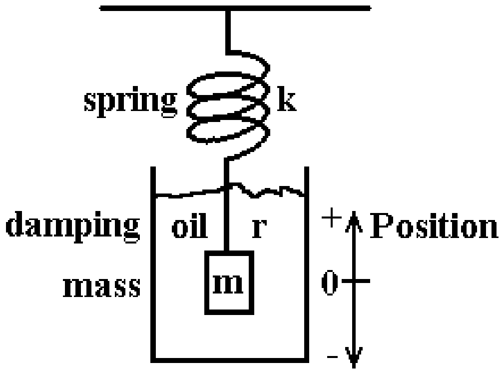
Kinetic Energy = $1/2 I\omega^2$

1.5 The Basic 2nd Order Mechanical System

A spring with constant k $Force = -ky$

A Mass with mass m

Some oil with constant damping R $Force = -Rv$



y is the signed displacement from equilibrium (at rest with y = 0).

Minus sign on spring term means force acts to restore mass to rest position.

Minus sign on damping term means force acts against motion, proportional to velocity.

$$\sum F = ma = -ky - Rv_y + mg = ma_y$$

assume gravity negligible:

$$\sum F = ma = -ky - Rv_y = ma_y$$

$$m\frac{d^2y}{dt^2} + R\frac{dy}{dt} + ky = 0$$

$$or \qquad \frac{d^2y}{dt^2} + \frac{R}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

Solutions:

Case 1: $(R/m)^2 - 4(k/m) < 0$ (Under damped)

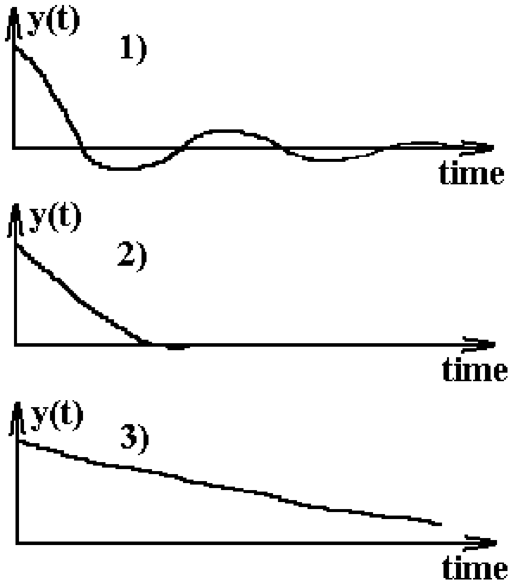
$$y = e^{-\alpha t}(c_1 sin(\beta t) + c_2 cos(\beta t))$$

Case 2: $(R/m)^2 - 4(k/m) = 0$ (Critically damped)

$$y = c_1 e^{-\alpha t} + c_2 t e^{-\alpha t}$$

Case 3: $(R/m)^2 - 4(k/m) > 0$ (Over damped)

$$y = c_1 e^{-\alpha_1 t} + c_2 e^{-\alpha_2 t}$$



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